Vertex functions for d-wave mesons in the light-front approach

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Abstract

While the light-front quark model (LFQM) is employed to calculate hadronic transition matrix elements, the vertex functions must be pre-determined. In this work we derive the vertex functions for all d-wave states in this model. Especially, since both of $^3D_1$ and $^3S_1$ are 1^{--} mesons, the Lorentz structures of their vertex functions are the same. Thus when one needs to study the processes where $^3D_1$ is involved, all the corresponding formulas for $^3S_1$ states can be directly applied, only the coefficient of the vertex function should be replaced by that for $^3D_1$. The results would be useful for studying the newly observed resonances which are supposed to be d-wave mesons and furthermore the possible 2S-1D mixing in ψ' with the LFQM.

PACS numbers: 12.39.Ki, 14.40.-n

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arXiv:1104.3996v3 [hep-ph] 4 Oct 2011

I. INTRODUCTION

In the hadronic physics the most tough job is to calculate the hadronic transition matrix elements which are fully governed by non-perturbative QCD. Because of lack of solid knowledge on non-perturbative QCD so far, one needs to invoke phenomenological models. Applications of such models to various processes have achieved relative successes so far. Among these models the light-front quark model(LFQM) is a relativistic model and has obvious advantages for dealing with the hadronic transitions where light hadrons are involved [1, 2]. The light-front wave function is manifestly Lorentz invariant and expressed in terms of fractions of internal momenta of the constituents which are independent of the total hadron momentum. This approach has been applied to many processes and thoroughly discussed in literatures [3–13]. Generally the results qualitatively coincide with experimental observation and while taking the error ranges into account (both experimental and theoretical), they can be considered to quantitatively agree with data.

However earlier researches with the LFQM only concern the s-wave and p-wave mesons whereas the higher orbital excited states have not been discussed yet. With the improvements of experimental facilities and rapid growth of database many new resonances have been observed and some of them are regarded as higher orbital excited states, for example $\psi(3770)$ and $\psi(4153)$ are suggested to be d-wave charmonia whose principal quantum numbers are respectively $n = 1$ and $n = 2$[14]. New analysis on $X(3872)$ indicates that it has a possible charmonium assignment $^1D_2$[14]. Moreover, a d-wave state $\Upsilon(1^3D_2)$ was observed by CLEO[15]. The situation persuades us to extend our scope to involve d-wave. It is generally believed that only the lattice theory indeed deals with the non-perturbative QCD effects from the first principle. So far, the lattice study is constrained by not only the computing abilities, but also the theory itself. Even so, remarkable progresses have been made on the two aspects. It is hoped that the lattice calculation will eventually solve all the problems on hadrons, such as the hadron spectra, wavefunctions, even the hadronic transition matrix elements. At present, the lattice calculation indeed shed some light on the wavefunctions[16–18]. For examples, the authors of Ref.[16] suggested a method to compute the spectra and wavefunctions of hadron excited states and applied their method to the $U(1)_{2+1}$ lattice gauge theory. Abada $et.\,al.$[17] study the pion light-cone wave function on the lattice by considering the three-point Green functions. On other aspects, for example, the readers who are interested in the unquenched lattice calculation on the charmonia whose total quantum numbers J^{PC} are determined simultaneously, are suggested to refer to those enlightening papers [19]. Some works about the radiative decays of charmonia are included in concerned references[20, 21]. More similar works can be found in most recent literatures[22–24]. In fact, due to the rapid progress of lattice calculations, people are more tempted to trust those results, but it by no means implies that we should abandon phenomenological models because those models are directly invented to manifest the physical mechanisms and

moreover, they are simpler and applicable in practice.

To evaluate the transition rate in the LFQM one needs to know the wave functions of parent and daughter hadrons. For any $^{2S+1}L_J$ state, its wavefunction is constructed as the corresponding spinors multiplying the so-called vertex function which should be theoretically derived. It is also noted the wavefunctions for s-wave and p-wave have been derived and their explicit forms are given in Ref.[7]. Following the same strategy we obtain the wavefunction for all d-wave states.

The traditional LFQM was employed to study the decay constants and form factors of weak decays [4–6], but to maintain the Lorentz covariance other contributions such as Z-diagram[5] or zero-mode[25–28] contributions must be included.

Thus a covariant LFQM[6] has been suggested which systematically includes the zero-mode contributions. In the traditional LFQM the constituent quarks in the bound state are required to be on their mass shells, nevertheless in the covariant LFQM approach the constituent quarks in the meson are off-shell while only the meson is on its mass shell. In this approach, one writes down the transition amplitudes where all quantities and the integrations maintain their four-dimensional forms, then integrates out the light-front momentum p^- in a proper way. While carrying out the contour integration the antiquark is enforced to be on its mass shell. The integrand in the remaining three-dimension integration reduces into a form where all quantities can be expressed in terms of the conventional wavefunctions. During this procedure, some extra contributions emerge comparing with the original scheme (see the text for details).

In this work after this introduction we derive the phenomenological vertex functions for d-wave in the conventional light-front approach in section II. Then in section III we present their forms in the covariant light-front approach. In section IV we discuss some formula for $^3D_1$ states and the section V is devoted to a brief summary.

II. VERTEX FUNCTIONS IN THE CONVENTIONAL LIGHT-FRONT APPROACH

Let us first derive the vertex functions in the conventional light-front approach.

In the conventional light-front approach a meson with the total momentum P and spin J can be written as [7]

$$
\begin{aligned}
|M(P, {}^{2S+1}L_J, J_Z)\rangle \quad &= \int \{d^3\tilde{p}_1\}\{d^3\tilde{p}_2\}2(2\pi)^3\delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \\
&\sum_{\lambda_1\lambda_2} \Psi_{LS}^{JJ_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2)|q_1(p_1, \lambda_1)\bar{q}_2(p_2, \lambda_2)\rangle,
\end{aligned}
\tag{1}
$$

where the flavor and color indices are omitted; q_1 and $\bar{q}_2$ correspond to the quark and antiquark in the meson and p_1 and p_2 are the on-shell light-front momenta of quark and

3

antiquark, $\mathbf{p}$ is the three-momentum $(\mathbf{p}_1 - \mathbf{p}_2)/2$ and we define

$$\tilde{p}_i = (p_i^+, p_{i\perp}), \ p_{i\perp} = (p_i^1, p_i^2), \ p_i^- = \frac{m^2 + p_{i\perp}^2}{p_i^+}, \ \{d^3\tilde{p}_i\} \equiv \frac{dp_i^+ d^2 p_{i\perp}}{2(2\pi)^3},$$

$$|q_1(p_1, \lambda_1)\bar{q}_2(p_2, \lambda_2)\rangle = b_{\lambda_1}^\dagger(p_1) d_{\lambda_2}^\dagger(p_2)|0\rangle.$$

The light-front momenta p_1 and p_2 are expressed via the variables p and x_i $(i = 1, 2)$ as

$$p_1^+ = x_1 P^+, \ p_2^+ = x_2 P^+, \ x_1 + x_2 = 1,$$
$$p_{1\perp} = x_1 P_\perp + p_\perp, \ p_{2\perp} = x_2 P_\perp - p_\perp. \tag{2}$$
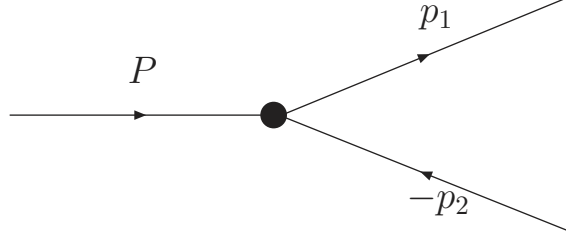

FIG. 1: The vertex for meson-quark-antiquark.

In the momentum representation, the wavefunction $\Psi_{LS}^{JJ_z}$ for the state $^{2S+1}L_J$ can be decomposed into the form

$$\Psi_{LS}^{JJ_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = \frac{1}{\sqrt{N_c}} \langle LS; L_z S_z | LS; JJ_z \rangle R_{\lambda_1 \lambda_2}^{SS_z}(x, p_\perp) \varphi_{LL_z}(x, p_\perp), \tag{3}$$

where $\varphi_{LL_z}(x, p_\perp)$ describes the relative momentum distribution of the quark (antiquark) in the meson and L is the orbital angular momentum between the constituents. $R_{\lambda_1 \lambda_2}^{SS_z}$ transforms a light-front helicity (λ_1, λ_2) eigenstate to a state with definite spin (S, S_z) and it is expressed as

$$R_{\lambda_1 \lambda_2}^{SS_z}(x, p_\perp) = \frac{1}{\sqrt{2}\tilde{M}_0(M_0 + m_1 + m_2)} \bar{u}(p_1, \lambda_1)(\bar{\slashed{P}} + M_0)\Gamma_S v(p_2, \lambda_2),$$
$$\Gamma_0 = \gamma_5 \quad (\text{for } S = 0),$$
$$\Gamma_1 = -\slashed{\epsilon}(S_z) \quad (\text{for } S = 1), \tag{4}$$

with $\bar{P} = p_1 + p_2$, $M_0^2 = \frac{m_1^2 + p_\perp^2}{x_1} + \frac{m_2^2 + p_\perp^2}{x_2}$ and $\tilde{M}_0 \equiv \sqrt{M_0^2 - (m_1 - m_2)^2}$. For more details, readers are suggested to refer Ref.[7].

4

In the LFQM, the harmonic oscillator wavefunctions are employed to describe the relative 3-momentum distribution of quark and antiquark in a meson. For d-wave the harmonic oscillator wavefunction is[29]

$$\varphi_{2L_z}(x, p_\perp) = \hat{\varepsilon}^{\mu\nu}(L_z) K_\mu K_\nu \frac{\sqrt{2}}{\beta^2} \varphi, \tag{5}$$

where $K = (p_2 - p_1)/2$ and φ is the harmonic oscillator wavefunction for s wave and its explicit expression is with

$$\varphi = 4(\frac{\pi}{\beta^2})^{3/4} \sqrt{\frac{dp_z}{dx_2}} \exp(-\frac{p_z^2 + p_\perp^2}{2\beta^2}),$$

$$p_z = \frac{x_2 M_0}{2} - \frac{m_2^2 + p_\perp^2}{2x_2 M_0}. \tag{6}$$

Substituting Eq.(3) into Eq.(2), we deduce the expression

$$\begin{aligned}
\Psi_{2S}^{JJ_z}(\tilde{p_1}, \tilde{p_2}, \lambda_1, \lambda_2) &= \frac{1}{\sqrt{N_c}} \langle 2S; L_z S_z | 2S; J J_z \rangle R_{\lambda_1 \lambda_2}^{SS_z}(x, p_\perp) \varphi_{2L_z}(x, p_\perp) \\
&= \frac{1}{\sqrt{N_c}} \frac{\varphi}{\sqrt{2} \tilde{M}_0 (M_0 + m_1 + m_2)} \bar{u}(p_1, \lambda_1)(\bar{\slashed{P}} + M_0)\Gamma_{(^{2s+1}D_J)} v(p_2, \lambda_2),
\end{aligned} \tag{7}$$

with

$$\Gamma_{(^3D_1)} = \sqrt{\frac{6}{5\beta^4}}[-\frac{\slashed{\hat{\varepsilon}}(J_z)}{3}(\frac{K \cdot \bar{P}^2}{M_0^2} - K^2) + \frac{\bar{\slashed{P}} K \cdot \bar{P} K \cdot \hat{\varepsilon}(J_z)}{M_0^2} - \slashed{K} K \cdot \hat{\varepsilon}(J_z)],$$

$$\Gamma_{(^1D_2)} = \sqrt{\frac{2}{\beta^4}} \hat{\varepsilon}^{\mu\nu}(J_z) K_\mu K_\nu \gamma_5,$$

$$\Gamma_{(^3D_2)} = \sqrt{\frac{4}{3\beta^4}} \hat{\varepsilon}^{\mu\nu}(J_z) \gamma_5 [\gamma_\nu \gamma_\mu \frac{(K \cdot \bar{P})^2 - M_0^2 K^2}{3M_0} - \gamma_\mu K_\nu (K \cdot \bar{P} - M_0 \slashed{K}) + K_\mu K_\nu],$$

$$\Gamma_{(^3D_3)} = \sqrt{\frac{2}{9\beta^4}} \hat{\varepsilon}^{\mu\nu\alpha}(J_z) \gamma_\beta (K_\mu K_\nu g_{\alpha\beta} + K_\mu K_\alpha g_{\nu\beta} + K_\alpha K_\nu g_{\mu\beta}), \tag{8}$$

where relations $\langle 2\,0; L_z\,0 | 2\,0; 2\,J_z \rangle = \hat{\varepsilon}^{*\mu\nu}(L_z)\hat{\varepsilon}_{\mu\nu}(J_z)$, $\langle 2\,1; L_z\,S_z | 2\,1; 1\,J_z \rangle = -\sqrt{\frac{3}{5}}\hat{\varepsilon}^{*\mu\nu}(L_z)\hat{\varepsilon}_{*\mu}(S_z)\hat{\varepsilon}_\nu(J_z)$, $\langle 2\,1; L_z\,S_z | 2\,1; 2\,J_z \rangle = i\sqrt{\frac{2}{3}}\epsilon^{\alpha\beta\mu\nu}\hat{\varepsilon}_{\alpha\omega}^*(L_z)\hat{\varepsilon}_\beta^*(S_z)\hat{\varepsilon}_{\mu\omega'}(J_z)g^{\omega\omega'}\frac{\bar{P}_\nu}{M_0}$ and $\langle 2\,1; L_z\,S_z | 2\,1; 3\,J_z \rangle = \frac{1}{3}\hat{\varepsilon}^{\mu\nu\alpha}(J_z)[\hat{\varepsilon}_{\mu\nu}^*(L_z)\hat{\varepsilon}_\alpha^*(S_z) + \hat{\varepsilon}_{\mu\alpha}^*(L_z)\hat{\varepsilon}_\nu^*(S_z) + \hat{\varepsilon}_{\nu\alpha}^*(L_z)\hat{\varepsilon}_\mu^*(S_z)]$ are used.

One can further simplify these wavefunctions in terms of the Dirac equation $\slashed{p}_1 u(p_1) = m_1 u(p_1)$ and $\slashed{p}_2 v(p_1) = -m_2 v(p_1)$, so that all scalar products of vectors are replaced by only M_0, m_1 and m_2 via a simple algebra, thus the wave function is

$$\Psi_{2S}^{JJ_z}(\tilde{p_1}, \tilde{p_2}, \lambda_1, \lambda_2) = \bar{u}(p_1, \lambda_1) h'_{(^{2s+1}D_J)} \Gamma'_{(^{2s+1}D_J)} v(p_2, \lambda_2), \tag{9}$$

where

$$h'_{(^3D_1)} = -\sqrt{\frac{1}{N_c}} \frac{1}{\sqrt{2}\tilde{M}_0} \frac{\sqrt{6}}{12\sqrt{5}M_0^2\beta^2}[M_0^2 - (m_1 - m_2)^2][M_0^2 - (m_1 + m_2)^2]\varphi,$$

$$h'_{(^1D_2)} = \sqrt{\frac{1}{N_c}} \frac{1}{\tilde{M}_0\beta^2}\varphi,$$

$$h'_{(^3D_2)} = \sqrt{\frac{1}{N_c}} \sqrt{\frac{2}{3}} \frac{1}{\tilde{M}_0\beta^2}\varphi,$$

$$h'_{(^3D_3)} = \sqrt{\frac{1}{N_c}} \frac{1}{3} \frac{1}{\tilde{M}_0\beta^2}\varphi, \tag{10}$$

and

$$\Gamma'_{(^3D_1)} = [\gamma_\mu - \frac{1}{w_{(^3D_1)}}(p_1 - p_2)_\mu]\hat{\varepsilon}^\mu,$$

$$\Gamma'_{(^1D_2)} = \gamma_5 K_\mu K_\nu \hat{\varepsilon}^{\mu\nu},$$

$$\Gamma'_{(^3D_2)} = \gamma_5[\frac{1}{w_{(^3D_2)}^a}\gamma_\nu\gamma_\mu + \frac{1}{w_{(^3D_2)}^b}\gamma_\mu K_\nu + \frac{1}{w_{(^3D_2)}^c}\gamma_\nu K_\mu + \frac{1}{w_{(^3D_2)}^d}K_\mu K_\nu]\hat{\varepsilon}^{\mu\nu},$$

$$\Gamma'_{(^3D_3)} = [K_\mu K_\nu(\gamma_\alpha + \frac{2K_\alpha}{w_{(^3D_3)}}) + K_\mu K_\alpha(\gamma_\nu + \frac{2K_\nu}{w_{(^3D_3)}}) + K_\alpha K_\nu(\gamma_\mu + \frac{2K_\mu}{w_{(^3D_3)}})]\hat{\varepsilon}^{\mu\nu\alpha}, \tag{11}$$

with

$$w_{(^3D_1)} = \frac{(m_1 + m_2)^2 - M_0^2}{2M_0 + m_1 + m_2},$$

$$w_{(^3D_2)}^a = \frac{-12M_0^2}{[M_0^2 - (m_1 + m_2)^2][M_0^2 - (m_1 - m_2)^2]},$$

$$w_{(^3D_2)}^b = -\frac{6M_0^2}{(2M_0 + m_1 + m_2)[M_0^2 - (m_1 - m_2)^2]},$$

$$w_{(^3D_2)}^c = \frac{6M_0^2}{(M_0 - m_1 - m_2)[M_0^2 - (m_1 - m_2)^2]},$$

$$w_{(^3D_2)}^b = \frac{M_0}{m_2 - m_1},$$

$$w_{(^3D_3)} = M_0 + m_1 + m_2. \tag{12}$$

It is interesting to ask where the QCD which definitely governs the physical processes, gets involved or how can one implant the QCD information into our calculation in the LFQM. In Ref.[30] the authors derived an effective Hamiltonian for bound states in the light-front frame based on the standard Lagrangian of QCD. The vertex function is the effective coupling between the bound state and its constituent quarks, thus as the wavefunction of the bound state is obtained the effective vertex function is in hand. After a long discussion about the
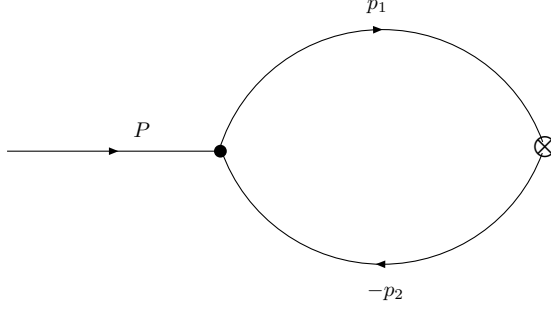
FIG. 2: The Feynman diagram for a meson annihilation.

Lorentz structure and the features of the dynamics of the vertex functions, they derive the vertex function which has exactly the form of Eq.(9) in our work. In Ref.[30], the radial wave function was obtained by solving the eigenvalue equation numerically. Obviously all QCD information (both short-distance and long-distance effects) is involved in the Hamiltonian and as well as in the solution. As argued in literature[5, 29], the solution can be well approximated by a Gaussian function with model parameters to be fixed by fitting data. Thus following Ref.[7] we choose a Gaussian wave function where the QCD information is included in the model parameter β.

We can apply these wavefunctions to deal with concrete physical processes. For example, when we calculate the rate of $^3D_1$ state annihilation through a vector current (in Fig.2), the transition amplitude is written as

$$
\begin{aligned}
\mathcal{A}_\mu^{conv} = & \ N_c \int d^3\tilde{p}_1 \Psi_{21}^{1\,J_z} \frac{\bar{v}(p_2, \lambda_2)}{\sqrt{p_2^+}} \gamma_\mu \frac{u(p_1, \lambda_1)}{\sqrt{p_1^+}} \\
= & \ N_c \int \frac{dx_1 d^2 p_\perp}{16\pi^3} \frac{h'_{3D_1}}{\sqrt{x_1 x_2}} \mathrm{Tr}[\Gamma'_{3D_1}(\not{p}_2 - m_2)\gamma_\mu(\not{p}_1 + m_1)] \\
= & \ N_c \int \frac{dx_1 d^2 p_\perp}{16\pi^3} \frac{h'_{3D_1}}{\sqrt{x_1 x_2}} \mathrm{Tr}\{[\gamma_\nu - \frac{(p_1 - p_2)_\nu}{w_{3D_1}}](\not{p}_2 - m_2)\gamma_\mu(\not{p}_1 + m_1)\}\hat{\varepsilon}^\nu. \quad (13)
\end{aligned}
$$

III. VERTEX FUNCTIONS IN THE COVARIANT LIGHT-FRONT APPROACH

Comparing with the conventional LFQM where both p_1 and p_2 are on their mass shells, in the covariant light-front approach the quark and antiquark are off-shell, but the total momentum $P = p_1 + p_2$ is the on-shell momentum of the meson, i.e. $P^2 = M^2$ where M is the mass of the meson. Obviously, the covariant LFQM is closer to the physical reality.

If one tries to obtain the covariant vertex functions based on an underlying principle, i.e. QCD, he should invoke a reasonable theoretical framework. To directly obtain the covariant

7

vertex functions in the 4-dimensional momentum space, the authors of Ref.[6] suggested to solve the Bethe-Salpeter (B-S) equations for the bound states [31, 32]. The kernel of the B-S equation includes the Coulomb piece which is induced by the one-gluon exchange as well as its higher-order corrections, and the confinement piece which incorporates the non-perturbative QCD but is not derivable so far. Genrally for solving the B-S equation, the instantaneous approximation is usually taken.

In the concrete calculations of the observable physical quantity in terms of the LFQM the final result is eventually reduced into an integration over the four-momentum. Fortunately, by doing so, we may not really need the explicit covariant wavefunctions defined in the four-momentum space. Namely, we try to reduce the integration into a simple form where only three-momentum wavefunctions remain by a mathematical manipulation, then we are able to relate the corresponding integrand to the conventional vertex function which is well defined in the three-momentum space.

Since the Lorentz structures of the covariant vertices are the same as that of the conventional vertex functions we rewrite these covariant vertex functions in Eq.(9) as

$$iH_{(^3D_1)}[\gamma_\mu - \frac{1}{W_{(^3D_1)}}(p_1 - p_2)_\mu]\varepsilon^\mu,$$

$$iH_{(^1D_2)}\gamma_5 K^\mu K^\nu \varepsilon^{\mu\nu},$$

$$iH_{(^3D_2)}\gamma_5[\frac{1}{W^a_{(^3D_2)}}\gamma^\omega\gamma^\mu + \frac{1}{W^b_{(^3D_2)}}\gamma^\mu K^\omega + \frac{1}{W^c_{(^3D_2)}}\gamma^\omega K^\mu + \frac{1}{W^d_{(^3D_2)}}K^\mu K^\omega]\varepsilon^{\mu\nu},$$

$$iH_{(^3D_3)}[K_\mu K_\nu(\gamma_\alpha + \frac{2K_\alpha}{W_{(^3D_3)}}) + K_\mu K_\alpha(\gamma_\nu + \frac{2K_\nu}{W_{(^3D_3)}}) + K_\alpha K_\nu(\gamma_\mu + \frac{2K_\mu}{W_{(^3D_3)}})]\varepsilon^{\mu\nu\alpha}, \quad (14)$$

where $H_{(^{2S+1}D_J)}$ and $W_{(^{2S+1}D_J)}$ are functions in the 4-dimensional space. Practically, the vertex function(s) is(are) included in a transition matrix element, for example, the amplitude of $^3D_1$ state annihilation via a vector current is written as

$$
\begin{aligned}
\mathcal{A}^{cov}_\mu &= -i^2 \frac{N_c}{16\pi^4} \int d^4p_1 \frac{H_{(^3D_1)}}{N_1 N_2} \text{Tr}\{[\gamma_\nu - \frac{(p_1 - p_2)_\nu}{W_{^3D_1}}](-\not{p}_2 + m_2)\gamma_\mu(\not{p}_1 + m_1)\}\varepsilon^\nu \\
&= \frac{-i^2 N_c}{16\pi^4} \int d^4p_1 \frac{H_{(^3D_1)}}{N_1 N_2} s\, \varepsilon^\nu, \quad (15)
\end{aligned}
$$

where $s = \text{Tr}\{[\gamma_\nu - \frac{(p_1-p_2)_\nu}{W_{^3D_1}}](-\not{p}_2 + m_2)\gamma_\mu(\not{p}_1 + m_1)\}$, $N_1 = p_1^2 - m_1^2 + i\epsilon$ and $N_2 = p_2^2 - m_2^2 + i\epsilon$. One first needs to integrate over p_1^- as discussed in Ref.[6, 7]. Integrating over p_1^- is completed by a contour integration where the antiquark is set on shell. Then the integration turns into

$$\frac{N_c}{16\pi^3} \int dx_1 d^2p_\perp \frac{h_{(^3D_1)}}{x_2 x_1 (M^2 - M_0^2)} \hat{s}\, \hat{\varepsilon}^\nu, \quad (16)$$

where $w_{(^3D_1)}$ and $\hat{\varepsilon}^\nu$ replace $W_{(^3D_1)}$ and ε^ν in Eq.(15) respectively.

Following Ref.[7] we have the relation

$$h_{(^3D_1)} = (M^2 - M_0^2)\sqrt{x_1 x_2}h'_{(^3D_1)}. \quad (17)$$

8

An additional factor $(M^2 - M_0^2)\sqrt{x_1 x_2}$ was introduced when comparing the decay constant f_P of pseudoscalar meson obtained in the two approaches as depicted in the appendix A of Ref.[7]. The legitimacy is guaranteed because the decay constant is free of zero mode contribution. Then the authors have applied the relation into the vertex functions for S and P waves. To show the reasonability of such replacement, we substitute Eq.(17) into Eq.(16) to obtain a new expression whose form is similar to the right side of Eq.(13). However, the trace in Eq. (16) involves the zero mode contribution which makes its form different from that in Eq.(13). Generally, after the contour integration over p_1^-, $h_{(2S+1}D_J)}$, $w_{(2S+1}D_J)}$ and $\hat{\varepsilon}$ replace $H_{(2S+1}D_J)}$, $W_{(2S+1}D_J)}$ and ε respectively with the following relation to the corresponding quantities of the conventional LFQM

$$h_{(2S+1}D_J)} = (M^2 - M_0^2)\sqrt{x_1 x_2}\, h'_{(2S+1}D_J)}. \tag{18}$$

Here we only concern the form of the covariant vertex function for the D-wave, including its Lorentz structure and coefficient, as well as its relations to the conventional vertex function. The details about the S- and P wave vertex functions were discussed in earlier literature[7]. When one needs to calculate a transition matrix in the covariant light-front quark model, he must know those vertex functions. Jaus has analyzed the case of the covariance of the transition matrix [6], and in his work, a general form of vertex function is used and the three-momentum conservation is automatically guaranteed. He [6] indicates that the general form of the vertex function h must be functions of $\hat{N}_i = x_i(M^2 - M_0^2)$ $(i = 1, 2)$. Obviously the function h adopted in this work coincides with this requirement.

IV. THE FORMULA FOR $^3D_1$ STATE

For a $J^{PC} = 1^{--}$ state, the the orbital momentum between the two constituents may be $L = 0$ (s-wave) or $L = 2$ (d-wave) and their total spin is 1 ($S = 1$). In Ref.[7] the authors gave the meson-quark-antiquark vertex for $^3S_1$ state as

$$iH_V[\gamma_\mu - \frac{1}{W_V}(p_1 - p_2)_\mu]. \tag{19}$$

Carrying out the contour integration over p_1^-, H_V and W_V turn into h_V and w_V

$$h_V = (M^2 - M_0^2)\sqrt{\frac{x_1 x_2}{N_c}}\frac{1}{\sqrt{2}\tilde{M}_0}\varphi,$$

$$w_V = M_0 + m_1 + m_2,$$

where the subscript V only refers to $^3S_1$ state.

The Lorentz structure of the vertex functions for $^3D_1$ and $^3S_1$ states are the same because they have the same quantum number J^{PC}. The difference between the s-wave and d-wave is included in the coefficient functions h_M and w_M.

The decay amplitude of an $^3S_1$ state via a vector current is proportional to [7]

$$A_\mu = -i^2 \frac{N_c}{(2\pi)^4} \int d^4 p_1 \frac{iH_V}{N_1 N_2} Tr\{\gamma_\mu(\not{p}_1 + m_1)[\gamma_\nu - \frac{(p_1 - p_2)_\nu}{W_V}](-\not{p}_2 + m_2)\}\hat\varepsilon^\nu, \qquad (20)$$

which is the same as Eq.(15), except $H_{^3D_1}$ and $W_{^3D_1}$ are replaced by H_V and W_V respectively. Integrating over p_1^- $H_{^3D_1}$, $W_{^3D_1}$, H_V and W_V reduce into $h_{^3D_1}$, $w_{^3D_1}$, h_V and w_V. The decay constant for $^3S_1$ state reads

$$f_V = \frac{N_c}{4\pi^3 M} \int dx_2 d^2 p_\perp \frac{h_V}{x_1 x_2 (M^2 - M_0^2)}[x_1 M_0^2 - m_1(m_1 - m_2) - p_\perp^2 + \frac{m_1 + m_2}{w_V} p_\perp^2], \quad (21)$$

so that one would obtain the decay constant of the $^3D_1$ state by replacing h_V and w_V by $h_{^3D_1}$ and $w_{^3D_1}$, thus it is

$$f_{^3D_1} = \frac{N_c}{4\pi^3 M} \int dx_2 d^2 p_\perp \frac{h_{^3D_1}}{x_1 x_2 (M^2 - M_0^2)}[x_1 M_0^2 - m_1(m_1 - m_2) - p_\perp^2 + \frac{m_1 + m_2}{w_{^3D_1}} p_\perp^2]. \quad (22)$$

In fact since the Lorentz structure of the vertex functions for $^3D_1$ and $^3S_1$ are the same all the formula for $^3D_1$ can be deduced from those for $^3S_1$. For example, the form factors f, g, a_+ and a_- of $P \to^3 D_1(V)$ decay can be obtained by simply replacing h_V and w_V of f, g, a_+ and a_- given in Ref.[6, 7].

With these formula we will be able to explore some new resonances of angular excited states, or furthermore to study the mixing of $2S - 1D$ which was proposed to explain the famous $\rho - \pi$ puzzle for ψ'[33–35] in this model.

V. A BRIEF SUMMARY

In this paper we deduce the vertex functions (or wave functions) for the d-wave in the conventional and covariant light-front quark model.

For the $^3D_1$ state the J^{PC} is 1^{--} and the Lorentz structure of its wave function is the same as that for the $^3S_1$ state so we obtain some useful formula for $^3D_1$ from the formula for $^3S_1$ given in Ref.[6, 7]. It is noted we just discuss the vertex functions $(i\Gamma_M)$ for the incoming meson whereas for the outgoing meson the corresponding vertex functions should be $i(\gamma_0 \Gamma_M^\dagger \gamma_0)$[7].

Since we adopt the Gaussian-type function for the radial part of the whole wavefunction instead of a solution obtained by solving the Schrödinger equation or the B-S equation, the simplification definitely brings up certain theoretical uncertainties, but as more data will be collected in the future, the more precise model parameter(s) will be determined and even the form of the wavefunction can be improved, thus we may do a better job along the line.

These vertex functions can be employed when one calculates the transition rates in this model. In the future we will study some concrete physical transitions where d-wave mesons

are involved in terms of these vertex functions. The results will be compared with data and the consistency would tell us the accuracy degree of the model and the derived vertex functions. Once the validity of the model is verified via some processes, we can further discuss some long-standing puzzles and help to identify new resonances which are continuously observed at BES and BELLE and elsewhere.

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